Preserver problems and reflexivity problems on operator algebras and on function algebras

PhD Thesis

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Debrecen, 2003

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In the present thesis we give an overview of our results from our dissertation.

Our dissertation consists of two parts. The first one presents our results on *preserver problems* concerning certain algebraic structures of linear operators or continuous functions. The second part is devoted to our results on *reflexivity problems* concerning certain algebras of functions. Both parts have separate introductions.

Some of our results presented here have appeared in our papers [23, 24, 28, 29, 69]. The other results will be published in our articles [25, 26, 27].

PART I
Some preserver problems

Linear preserver problems concern the question of determining all linear maps on an algebra which leave invariant a given subset, function or relation defined on the underlying algebra. The study of linear preserver problems on matrix algebras represents one of the most active research areas in matrix theory (see e.g. the survey papers [48, 49]). In the last decades considerable attention has also been paid to the infinite dimensional case, i.e. to preserver problems on operator algebras, and the investigations have resulted in several important results (see e.g. the survey paper [14]).

In what follows we mention three of the main groups of linear preserver problems on operator algebras. For brevity, we shall sometimes write **LPPs** for the expression **linear preserver problems**.

The *first group* of LPPs is concerned with the characterization of linear transformations on a linear space of bounded linear operators which preserve a certain given *function*. A well-known example of this kind of LPPs is Frobenius's result [22] from 1897. He gave the general form of all determinant preserving linear maps on a matrix algebra. This result is commonly considered as the first one on linear preserver problems. Another important example is the result of Jafarian and Sourour [37], where they described the general form of surjective linear transformations on the algebra of all bounded linear operators on a Banach space which preserve the spectrum of the operators.

The *second group* of LPPs deals with the characterization of linear transformations which preserve a certain *subset* of operators. As an example we mention Kaplansky's famous question [42] on invertibility preserving maps. Namely, he asked whether every surjective linear transformation between Banach algebras which preserves invertibility in one direction is a Jordan homomorphism. In such generality the answer turned out to be negative, but his question, modified with the additional assumption of the semi-simplicity of the underlying Banach algebras, is still open and is one of the most important unsolved preserver problems.

The *third group* of LPPs concerns the characterization of linear transformations which preserve a certain *relation* between operators. An important example is the problem of preserving commutativity, which topic is still an active research area (cf. [77, 78]).

In several cases LPPs on matrix algebras or on operator algebras can be reduced to linear preserver problems which concern rank (see e.g. [37, 87, 91]). Therefore, it is not surprising that there is a vast literature on such problems. We mention just two important finite-dimensional results here: Beasley's result [9] on rank-k preserving linear maps and Loewy's result [50] on rank-k non-increasing linear maps. With regard to the infinite-dimensional case, i.e. to preserver problems on operator algebras, the particular cases of preserving rank-1 operators or preserving operators with rank at most 1 have been treated in the papers [36, 79].

In **Chapter 2** we characterize the rank-k non-increasing linear maps, the rank-k preserving linear maps, and the corank-k preserving linear maps on the algebra of all bounded linear operators on a Hilbert space under a mild continuity condition, and we unify and extend the results mentioned above. (The problem of corank-k preservers occurs obviously in the infinite-dimensional case only.) Below we present only one of the theorems of Chapter 2. As one can see in the dissertation, the further preservers under consideration are of similar forms.

Theorem 2.2 *Let k be a positive integer and H a Hilbert space. Assume that $\phi : B(H) \to B(H)$ is a rank-k preserving linear map which is weakly continuous on norm bounded sets. Assume also that the image of ϕ is not contained in $B_k(H)$, where $B_k(H)$ denotes the set of all operators of rank k. Then there exists an injective operator $A \in B(H)$ and an operator $B \in B(H)$ with dense image such that either $\phi(T) = ATB$ for all $T \in B(H)$, or $\phi(T) = AT^{tr}B$ for all $T \in B(H)$, where T^{tr} denotes the transpose of T relative to an arbitrary but fixed orthonormal basis of H.*

The content of Chapter 2 was published in our paper [29].

The concept of *linear preserver problems* has so far meant investigations on matrix algebras and on operator algebras, but similar questions can obviously be raised on *arbitrary algebras*. In **Chapter 3** we consider LPPs concerning *function algebras*, in which case the main LPPs considered before have been the characterizations of linear bijections preserving some given norm, or preserving disjointness of the support. We refer to [1, 21, 33, 38, 93, 94] for some of the important and relatively recent papers on such problems. For convenience, we introduce some notation. Let X be a locally compact Hausdorff space and let $C_0(X)$ denote the algebra of all continuous complex valued functions on X which vanish at infinity. The linear bijections of $C_0(X)$ preserving the sup-norm are determined in the famous Banach-Stone theorem. Besides the sup-norm, one of the most natural possibilities to measure a function is to consider the diameter of its range. In Chapter 3 we characterize all the linear bijections of $C_0(X)$ which *preserve the diameter of the range*, and give a unification of the contents of our papers [23] and [28]. Namely, we prove the following theorem.

Theorem 3.1 *Let X be a first countable locally compact Hausdorff space, and let X_0 denote $X \cup \{\infty\}$ if X is not compact, and X if X is compact. Then X_0, endowed with a topology in a natural way, is a compact Hausdorff space.*

If X is compact, then a bijective linear map $\phi : C_0(X) \to C_0(X)$ is diameter preserving if and only if there exists a complex number τ of modulus 1, a homeomorphism $\varphi : X \to X$ and a linear functional $t : C_0(X) \to \mathbb{C}$ with $t(1) \neq -\tau$ such that ϕ is of the form

$$\phi(f) = \tau \cdot f \circ \varphi + t(f)1 \qquad \big(f \in C_0(X)\big). \tag{3.1}$$

If X is not σ-compact, then a bijective linear map $\phi : C_0(X) \to C_0(X)$ is diameter preserving if and only if there exists a complex number τ of modulus 1 and a homeomorphism $\varphi : X \to X$ such that ϕ is of the form

$$\phi(f) = \tau \cdot f \circ \varphi \qquad \big(f \in C_0(X)\big). \tag{3.2}$$

If the space X is σ-compact but not compact, then a bijective linear map $\phi : C_0(X) \to C_0(X)$ is diameter preserving if and only if there exists a complex number τ of modulus 1 and a homeomorphism $\varphi : X_0 \to X_0$ such that ϕ is of the form

$$\phi(f) = \tau \cdot f \circ \varphi - \tau f\big(\varphi(\infty)\big)1 \qquad \big(f \in C_0(X)\big), \tag{3.3}$$

where $f(\infty) = 0$ for every $f \in C_0(X)$.

Up to now we have considered *linear* preserver problems. It is clear that preserver problems can be raised *without assuming any kind of linearity.* We may consider transformations on some algebraic structures which preserve some property, quantity, relation, etc. There are a great number of results in mathematics which can be interpreted as preserver problems in this general sense. We mention the very simple example of the isometries of a metric space: the isometries can be viewed as transformations which preserve distance. Another example of preserver problems of this general kind is Wigner's famous unitary-antiunitary theorem, which we treat in our dissertation in detail. The next theorem presents one of its several formulations which characterizes the transformations on a Hilbert space preserving the absolute value of the inner product of any pair of vectors.

Theorem 4.1 *Let H be a complex Hilbert space and $T : H \to H$ an arbitrary function. Then*

$$|\langle Tx, Ty\rangle| = |\langle x, y\rangle| \tag{4.1}$$

holds for any $x, y \in H$ if and only if there exists a function $\varphi : H \to \mathbb{C}$, $|\varphi| = 1$ and a linear or conjugate linear isometry $U : H \to H$ such that

$$T = \varphi \cdot U. \tag{4.2}$$

This result is one of the most important theorems concerning the probabilistic aspects of quantum mechanics.

Several different proofs have been given for Wigner's fundamental theorem mentioned above. In **Chapter 4** we present a further, elementary proof which is based on a completely new approach. Our basic idea is as follows. We pick an orthonormal basis in the Hilbert space H, and first we prove that there exist φ and U (as in Theorem 4.1) for the set F of all vectors with *real coordinates.* To see this, we show that on an arbitrary subset $G \subseteq F$, the elements of which are not orthogonal to a given vector, our transformation T is of the form (4.2) with φ and U depending on G. To prove this, we consider all the subsets of $G \times G$ for the elements of which (4.2) holds with (not necessarily the same) adequate φ and U, and we show that those subsets satisfy the conditions of *Zorn's lemma.* So there is a maximal subset in $G \times G$ with this property, which turns out to be the whole $G \times G$. Then it is easy to show that there are φ and U for which T is of the form (4.2) on the *whole Hilbert space.*

Wigner's theorem has been generalized in (at least) three directions. *First*, Uhlhorn [92] generalized Wigner's result by requiring only the preservation of *orthogonality* instead of that of the absolute value of the inner product, and he was able to achieve the same conclusion for the case in which the underlying space is at least 3 dimensional. Uhlhorn's result has a serious impact in physics. *Secondly*, Bargmann [7] and Sharma and Almeida [89] obtained results similar to Wigner's *without the assumption of bijectivity.* As for the *third* direction, we recall that sometimes Wigner's theorem is formulated as the characterization of bijections of the set of all 1-dimensional subspaces of a Hilbert space which preserve the angle between those subspaces. Molnár [65] extended Wigner's result in this respect to transformations on the set of all *n-dimensional subspaces* (n being fixed) which preserve the so-called principle angles between the subspaces. (For other generalizations of Wigner's theorem see e.g. [55, 57, 59, 62]). In **Chapter 5** we extend Wigner's theorem in *all the three directions* mentioned above, by obtaining results on the structure of orthogonality preserving transformations on the set of all n-dimensional subspaces of a Hilbert space under various conditions. Namely, we prove the following theorem.

Theorem 5.1 *Let H be a Hilbert space and $n \in \mathbb{N} \cup \{\infty\}$ with*

$$(5.1) \qquad \begin{cases} \dim H > 2n & \text{if } n \in \mathbb{N}, \\ \dim H = \infty & \text{if } n = \infty, \end{cases}$$

and let $\phi : H_n \to H_n$, where H_n denotes the set of all n-dimensional closed linear subspaces of H, which are also of infinite codimension if $n = \infty$.

If $\phi : H_n \to H_n$ is ***surjective****, then ϕ preserves orthogonality in both directions if and only if there exists a unique bijection $\psi : H_1 \to H_1$ which preserves orthogonality in both directions, and for any $K \in H_n$ we have*

$$(5.2) \qquad \phi(K) = \operatorname{span}\{\psi(X) | X \in H_1, X \subseteq K\},$$

where span *denotes the generated linear subspace.*

Thus, by Uhlhorn's theorem, if $n \in \mathbb{N} \cup \{\infty\}$ is such that (5.1) *holds and $\phi : H_n \to H_n$ is surjective, then ϕ preserves orthogonality in both directions if and only if there exists a unitary or antiunitary operator U on H such that for any $K \in H_n$ we have*

$$(5.3) \qquad \phi(K) = U(K).$$

If H is ***finite dimensional****, then ϕ preserves orthogonality in both directions (surjectivity is not assumed) if and only if there exists a unique transformation $\psi : H_1 \to H_1$ which preserves orthogonality in both directions and for any $K \in H_n$* (5.2) *holds. Moreover, if ϕ preserves principal angles then ψ also preserves angles, thus in this case ϕ is of the form* (5.3) *with a unitary or antiunitary operator U on H.*

PART II
Some reflexivity problems on function algebras

In the **second part** of the dissertation we deal with the problem of ***reflexivity*** of the automorphism group and the isometry group of certain algebras of functions. The study of reflexive linear subspaces of the algebra $B(H)$ of all bounded linear operators on a Hilbert space H represents one of the most active research areas in operator theory (see [30] for a nice general view of reflexivity of this kind). In the last decades, similar questions concerning certain important sets of transformations acting on Banach algebras rather than on Hilbert spaces have also attracted considerable attention. The initiators of the research in this direction are Kadison, Larson and Sourour. Kadison [41] studied *local derivations* from a von Neumann algebra $\mathcal{R}$ into a dual $\mathcal{R}$-bimodule $\mathcal{M}$. He called a continuous linear map from $\mathcal{R}$ into $\mathcal{M}$ a local derivation, if it agrees with a derivation at each point in the algebra $\mathcal{R}$ (the derivation may differ from point to point). This investigation of Kadison was motivated by some problems concerning the Hochschild cohomology of operator algebras. The main result, Theorem A, in [41] states that in the above setting, every local derivation is a derivation. Independently, Larson and Sourour [46] proved that the same conclusion holds for the local derivations of $B(X)$ (the definition is clear), where X is a Banach space. Since then, a considerable amount of work has been done concerning local derivations of various algebras. See, for example, [11, 18, 32, 40, 76, 85, 95, 97, 98, 99, 100]. Besides derivations, there are at least two further very important classes of transformations on operator algebras which certainly deserve attention. Namely, the group of automorphisms and the group of surjective isometries. Larson [45, Some concluding remarks (5), p. 298] initiated the study of *local automorphisms* (the definition is self-explanatory) of Banach algebras. In his joint paper with Sourour [46] that we have already mentioned they proved that if X is an infinite dimensional Banach space, then every surjective local automorphism of $B(X)$ is an automorphism (see also the paper [11] of Brešar and Šemrl). For a separable infinite dimensional Hilbert space H, Brešar and Šemrl [12] showed that the above conclusion holds without the assumption of surjectivity, i.e. every local automorphism of $B(H)$ is an automorphism. For further results on local automorphisms, we refer to [72, 85]. The common feature of all those results is that they show that the local derivations, local automorphisms, local isometries, etc. of the underlying structures are (global) derivations, automorphisms, isometries, etc., respectively. Clearly, this is a remarkable property of the underlying structure. As for function algebras, results of this kind were obtained by Cabello Sánchez and Molnár [84], and by Molnár and Zalar [71].

We now define our concept of *reflexivity*. Let X be a Banach space (in fact, in the cases which we are interested in, X is usually a Banach algebra) and for any subset $\mathcal{E} \subset B(X)$ let

$$\mathrm{ref}_{alg}\,\mathcal{E} = \{T \in B(X) : Tx \in \mathcal{E}x \text{ for all } x \in X\},$$
$$\mathrm{ref}_{top}\,\mathcal{E} = \{T \in B(X) : Tx \in \overline{\mathcal{E}x} \text{ for all } x \in X\},$$

where bar denotes norm-closure. The above sets are called the *algebraic reflexive closure* and the *topological reflexive closure* of $\mathcal{E}$, respectively. The collection $\mathcal{E}$ of transformations is called *algebraically reflexive* if $\mathrm{ref}_{alg}\,\mathcal{E} = \mathcal{E}$, and *topologically reflexive* if $\mathrm{ref}_{top}\,\mathcal{E} = \mathcal{E}$.

In this terminology, the main result in [12] can be reformulated by saying that the automorphism group of $B(H)$ is algebraically reflexive. Similarly, Theorem 1.2 in [46] states that the Lie algebra of all generalized derivations on $B(X)$ is algebraically reflexive. For some further results on the algebraic reflexivity of the automorphism and isometry groups we refer to [61, 71, 84].

Obviously, topological reflexivity is a stronger property than algebraic reflexivity. Shulman [90] showed that the derivation algebra of any C^*-algebra is topologically reflexive. Hence, not only the local derivations are derivations in this case, but every bounded linear map which agrees with the limit of some sequence of derivations at each point (this sequence may differ from point to point) is a derivation. For the topological reflexivity of derivation algebras, automorphism groups and isometry groups, we refer to [8, 39, 54, 56].

For the automorphism group or the isometry group of C^*-algebras, such a general result as in [90] does not hold. If $\mathcal{A}$ is a Banach algebra, then denote by $\mathrm{Aut}(\mathcal{A})$, $\mathrm{Aut}^*(\mathcal{A})$ and $\mathrm{Iso}(\mathcal{A})$ the group of automorphisms (i.e. multiplicative linear bijections), the group of $*$-automorphisms and the group of surjective linear isometries of $\mathcal{A}$, respectively. Now, if X is an uncountable discrete topological space, then it is not difficult to verify that the groups $\mathrm{Aut}(C_0(X))$ and $\mathrm{Iso}(C_0(X))$ of the C^*-algebra $C_0(X)$ of all continuous complex valued functions on X vanishing at infinity are not reflexive even algebraically. With regard to topological reflexivity, there are even von Neumann algebras whose automorphism and isometry groups are not topologically reflexive. For example, Batty and Molnár [8] showed that the infinite dimensional commutative von Neumann algebras acting on a separable Hilbert space have this nonreflexivity property. However, Molnár [54] proved that if H is a separable infinite dimensional Hilbert space, then $\mathrm{Aut}(B(H))$ and $\mathrm{Iso}(B(H))$ are topologically reflexive. For an interesting result we refer to [61], where Molnár studied the reflexivity of the automorphism and isometry groups of C^*-algebras appearing in the famous Brown-Douglas-Fillmore theory, which are extensions of the C^*-algebra of all compact operators on H by commutative separable unital C^*-algebras. He proved that the groups Aut and Iso are algebraically reflexive in the case of every such extension, but, for example, in the probably most important case of extensions by $C(\mathbb{T})$ (i.e. the so called Toeplitz extensions) those groups are *not* topologically reflexive.

In **Chapter 6** we deal with the reflexivity of the automorphism group and the isometry group of the suspension of $B(H)$. The concept of the suspension of a C^*-algebra plays a very important role in the K-theory of operator algebras. If $\mathcal{A}$ is a C^*-algebra then its suspension is the C^*-tensor product $C_0(\mathbb{R}) \otimes \mathcal{A}$, which is well-known to be isomorphic to $C_0(\mathbb{R}, \mathcal{A})$, the algebra of all continuous functions from $\mathbb{R}$ into $\mathcal{A}$ which vanish at infinity. We know that the automorphism group and the isometry group of $B(H)$ are topologically reflexive [54]. We show that $\mathrm{Aut}(C_0(\mathbb{R}))$ and $\mathrm{Iso}(C_0(\mathbb{R}))$ are algebraically (but not topologically) reflexive. In Chapter 6 we obtain several results, the following corollary of which can be considered as the main result of Chapter 6.

Corollary 6.5 *The automorphism and isometry groups of the suspension of $B(H)$ are algebraically reflexive.*

The content of Chapter 6 was published in our paper [69]. The referee of the manuscript put the question whether it is possible to describe the topological reflexive closures of $\mathrm{Aut}(C_0(\mathbb{R}) \otimes B(H))$ and $\mathrm{Iso}(C_0(\mathbb{R}) \otimes B(H))$. **Chapter 7** is devoted to answer this question. Among others, we obtain the following result.

Corollary 7.2 *Let $\phi : C_0(\mathbb{R}, B(H)) \to C_0(\mathbb{R}, B(H))$ be a linear map. We have $\phi \in \mathrm{ref}_{top} \mathrm{Iso}(C_0(\mathbb{R}, B(H)))$ resp. $\phi \in \mathrm{ref}_{top} \mathrm{Aut}^*(C_0(\mathbb{R}, B(H)))$, if and only if there exists an open interval $U \subseteq \mathbb{R}$, a surjective, monotone, continuous function $\varphi : U \to \mathbb{R}$, and $\tau : U \to \mathrm{Iso}(B(H))$ resp. $\tau : U \to \mathrm{Aut}^*(B(H))$, such that ϕ is of the form*

$$\phi(f)(y) = \begin{cases} \tau(y)\big(f\big(\varphi(y)\big)\big) & \text{if } y \in U, \\ 0 & \text{if } y \in \mathbb{R} \setminus U \end{cases} \tag{7.1}$$

for any $f \in C_0(\mathbb{R}, B(H))$. Moreover, if ϕ is of the form (7.1), then τ is strongly continuous.

In the concept of local derivations, local automorphisms, etc. we supposed that the transformations under consideration are linear and they equal a derivation, automorphism, etc., respectively, at every single point of the underlying algebra. If we drop the condition of linearity, it is easy to see that the obtained concept is so general (because the assumption is so weak) that it is practically useless. Motivated by the result of Kowalski and Slodkowski [44] on a non-linear characterization of the characters of commutative Banach algebras, Šemrl [88] introduced the concept of ***2-locality***. For example, we say that a (not necessarily linear) transformation ϕ of a Banach algebra is called a 2-local automorphism, if for any pair x, y of points in the Banach algebra under consideration we have an automorphism $\phi_{x,y}$ (depending on x and y) such that $\phi(x) = \phi_{x,y}(x)$ and $\phi(y) = \phi_{x,y}(y)$. The definition of 2-local derivations, 2-local isometries, etc. are similar. Šemrl [88] proved that every 2-local automorphism of $B(H)$ (H being an infinite dimensional separable Hilbert space) is an automorphism, and every 2-local derivation of $B(H)$ is a derivation.

This concept of 2-locality has the advantage that it can be considered in relation with any algebraic structure as we do not assume any kind of linearity. Clearly, it is a remarkable property of the underlying algebraic structure if its 2-local automorphisms, 2-local (surjective linear) isometries, etc. are (global) automorphisms, isometries, etc., respectively. In fact, this means that the automorphisms, isometries, etc. are determined by their local actions on the 2-point subsets. For some recent results on 2-local derivations, 2-local automorphisms and 2-local isometries we refer to [6, 35, 43, 64, 66, 67, 70].

Molnár [66] studied 2-local isometries of operator algebras. He proved that every such transformation of a C^*-subalgebra of $B(H)$ which contains the compact operators and the identity is a (surjective linear) isometry. Moreover, he raised the problem of considering similar questions concerning function algebras. In **Chapter 8** we obtain such a result for one of the most important types of function algebras, namely for $C_0(X)$. We prove the following statement.

Theorem 8.2 *If X is a first countable σ-compact Hausdorff space then every 2-local isometry of $C_0(X)$ is a (surjective linear) isometry.*

The content of Chapter 8 appeared in our paper [24].

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List of Publications Treated in the Thesis

1. M. Győry and L. Molnár, Diameter preserving bijections of $C(X)$,

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2. M. Győry, L. Molnár and P. Šemrl, Linear rank and corank preserving maps on $B(H)$ and an application to *-semigroup isomorphisms of operator ideals,

   Linear Alg. Appl. **280** (1998), 253–266.

3. L. Molnár and M. Győry, Reflexivity of the automorphism and isometry groups of the suspension of $B(H)$,

   J. Funct. Anal. **159** (1998), 568-586.

4. M. Győry, Diameter preserving bijections of $C_0(X)$,

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5. M. Győry, 2-local isometries of $C_0(X)$,

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6. M. Győry, On the transformations of all n-dimensional subspaces of a Hilbert space preserving orthogonality,

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7. M. Győry, On the reflexivity of the isometry groups of the suspension of $B(H)$,

   Studia Math., to appear.

8. M. Győry, A new elementary proof for Wigner's theorem,

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List of Talks at International Conferences

1. Diameter preserving linear bijections of $C(X)$,

   17th International Conference on Operator Theory,

   Temesvár (Romania), 1998

2. Diameter preserving linear bijections of $C_0(X)$,

   Numbers, Functions, Equations '98 International Conference,

   Noszvaj (Hungary), 1998

3. Reflexivity of the automorphism and isometry groups of some operator algebras,

   2nd Workshop on Functional Analysis and its Applications in Mathematical Physics and Optimal Control,

   Nemecka (Slovakia), 1999

4. On the 2-local isometries of $C_0(X)$,

   38th International Symposium on Functional Equations,

   Noszvaj (Hungary), 2000

5. Transformations on the set of all n-dimensional subspaces of a Hilbert space preserving orthogonality,

   3rd Workshop on Functional Analysis and its Applications in Mathematical Physics and Optimal Control,

   Nemecka (Slovakia), 2001

6. Transformations on the set of all n-dimensional subspaces of a Hilbert space preserving orthogonality,

   8th International Conference on Functional Equations and Inequalities,

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